

# Dynamics of double-well Bose-Einstein Condensates subject to external Gaussian white noise

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Dynamical properties of the Bose-Einstein condensate in double-well potential subject to Gaussian white noise are investigated by numerically solving the time-dependent Gross-Pitaevskii equation. The Gaussian white noise is used to describe influence of the random environmental disturbance on the double-well condensate. Dynamical evolutions from three different initial states, the Josephson oscillation state, the running phase and  $\pi$ -mode macroscopic quantum self-trapping states are considered. It is shown that the system is rather robust with respect to the weak noise whose strength is small and change rate is high. If the evolution time is sufficiently long, the weak noise will finally drive the system to evolve from high energy states to low energy states, but in a manner rather different from the energy-dissipation effect. In presence of strong noise with either large strength or slow change rate, the double-well condensate may exhibit very irregular dynamical behaviors.

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## I. INTRODUCTION

The atomic Bose-Einstein condensate (BEC) trapped in double-well potentials builds up bosonic Josephson junction (BJJ) [1–5]. Since it exhibits abundant quantum properties in comparison to condensate in a single trap, the double-well condensate has already been intensively investigated theoretically in the last few years. As a BJJ, the double-well condensate can not only display dc, ac Josephson effects and the Shapiro effect. It also exhibits an quantum nonlinear effect, named the macroscopic quantum self-trapping (MQST) [3]. On the other hand, quantum fluctuation is believed to give rise to fascinating influence on above dynamical behaviors [6–12], such as collapse and revival of quantum oscillations [6, 7], disappearance of coherence [8, 9], and destruction of the self-trapped state [10].

Experimentally the Josephson tunneling and MQST in a single BJJ have been observed in 2005 [13]. Since then, more progress has been made in studying static, thermal and dynamical properties of the double-well condensate. Experimental investigation of thermal induced phase fluctuations has been reported [14]. Measurements of the ac and dc Josephson effects in BJJ have already been realized [15]. The BJJ system has also been used to perform interference-fringe experiments [16] and to investigate the crossover from Josephson dynamics to hydrodynamics [17]. In above theoretical studies, the double-well condensate is mainly treated as an isolated system, but actually it is coupled to certain thermal cloud and subject to environmental distortions in experiments. Dissipation and noise effects play important roles in understanding properties of BJJs.

The dissipative effect has been studied by several groups [18–23]. It is suggested that the MQST state

can be destroyed by energy dissipation [18–20]. Possible decoherence caused by dissipation is also discussed [21, 22]. It is also shown that dissipation could lead to enhancement of coherence under specific conditions [23].

Here we consider a kind of noise effect on the double-well condensate. Noise can be classified as “internal noise” or “external noise” with respect to its origins [24]. Internal noise comes from the inside fluctuations of the system, including quantum and thermal fluctuations, or from the exchange symmetry of identical particles [25, 26]. On the other hand, “external noise” is brought about by fluctuations which are not “self-originating”. It is induced by the coupling between the system and its environment. For cold atomic systems, external noise may originate from the magnetic field, laser beams, or other externally applied random driving field. The present work will focus on this kind of external noise.

A number of theoretical works have been devoted to the influence of external noise on the double-well condensate [23, 27, 28]. The noise-induced dephasing [27] and phase decoherence [28] have been predicted. These works based on the two-mode approximation mainly discussed the phase noise. The phase noise is introduced by coupling a stochastic fluctuations either to the tunnel amplitude or to the number-imbalance operator. In their treatment, spatial information of the external noise is ignored.

The present paper deals with noise due to fluctuations of magnetic field or the optical potential. We shall consider the time- and space-dependent characteristics of the external noise by solving the time-dependent Gross-Pitaevskii (GP) equation numerically. In Sect. II, we describe a noise model in which the noise is simulated by spatially distributed stochastic potentials. The concept of Gaussian white noise and the algorithm used to solve the time-dependent GP equation for the double-well condensate are briefly described. We present in Sect. III the obtained results and discuss the influence of noise on the dynamical behaviors of system. A brief summary is given

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in the last section.

## II. THE MODEL AND METHOD

We consider the BEC confined in a double-well potential is composed of atoms of mass  $m$  with weakly repulsive interaction. Dynamics of the double-well condensate is obtained by solving the time-dependent GP equation numerically[29, 30]. We formulate the external noise by an additional potential of stochastic strength fluctuation in the space.

The external noise is modeled as the Gaussian white noise potential  $V_n(z, t)$ , which is space- and time-dependent stochastically. So  $V_n(z, t)$  satisfies the following equations,

$$\langle V_n(z_i, t) \rangle = 0, \quad (1)$$

and

$$\langle V_n(z_i, t) V_n(z_j, t') \rangle = 2D_0 \delta(t - t') \delta(z_i - z_j), \quad (2)$$

where  $\langle \rangle$  denotes averaging over both the space and the time. The Dirac delta function in the correlation formula makes sure  $V_n(z, t)$  is “white” noise,  $D_0$  is the fluctuation amplitude of noise potential. Apparently,  $D_0$  characterizes the strength or intensity of noise and it can be controlled experimentally.

The noise correlation function in time at a given position is defined as  $g(t - t') = \langle V_n(z_i, t) V_n(z_i, t') \rangle_T - \langle V_n(z_i, t) \rangle_T^2 \sim \delta(t - t')$  where  $\langle \rangle_T$  means that the average is just taken over time. Under this definition,  $\langle V_n(z, t) \rangle_T$  must be zero at all given positions. In numerical calculations,  $V_n(z, t)$  is produced as a time-ordered series,  $V_n(z, t_i)$ , so the time average is performed as  $\langle V_n(z, t) \rangle_T = \frac{\sum_{i=1}^M V_n(z_i, t_i)}{M}$ .  $M$  is actually a finite number, and  $M\Delta t$  is the total time interval with  $\Delta t = t_{i+1} - t_i$  being the step interval of the noise potential series. In our calculations,  $M = 100$  is already enough to makes sure  $\langle V_n(z, t) \rangle_T = 0$ . Therefore  $\Delta t$  can be used to evaluate the velocity of noise. Faster noise corresponds to smaller  $\Delta t$ .

The general three dimensional time-dependent GP equation provides an exact and fundamental description for our research, which can be formulated as

$$i\hbar \frac{\partial \psi(\mathbf{r}; t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + gN|\psi(\mathbf{r}; t)|^2 \right] \psi(\mathbf{r}; t), \quad (3)$$

where  $\psi(\mathbf{r}; t)$  is the macroscopic wave function at position  $\mathbf{r}$  and time  $t$ ,  $g = 4\pi\hbar^2 a/m$  is the nonlinear interaction with  $a$  being the  $s$ -wave scattering length. The external potential  $V(\mathbf{r}; t)$  consists of the double well  $V_{dw}(\mathbf{r})$  and the noise potential  $V_n(\mathbf{r}; t)$ .  $V_{dw}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 r_x^2 + \omega_y^2 r_y^2 + \omega_z^2 r_z^2) + V_b \exp(-r_z^2/q_0^2)$  fixes the double-well configuration, where  $\omega_i (i = x, y, z)$  is the trap frequencies in the direction of  $i$  [12, 13] and  $V_b$  is the barrier height. Experimentally, the Bose-Einstein condensates with repulsive

interaction in a Quasi-1D symmetric double-well potential can be achieved by splitting one cigar-shaped atomic cloud into two separated aligned cigars by a laser beam.

Set the double wells being along the  $z$  direction and in  $x$  and  $y$  directions the strong confinement is exerted ( $\omega_x = \omega_y \gg \omega_z$ ). Thus the original three dimensional condensates shall be reduced into a quasi one dimensional cigar BECs and Eq. (3) is reduced into a one-dimensional one. By scaling the length and energy as  $l_z = \sqrt{\hbar/(m\omega_z)}$  and  $\hbar\omega_z$  respectively, the dimensionless parameters are simplified as  $z = r_z/l_z$ ,  $\tau = t\omega_z/2$  and  $\beta = \sqrt{\omega_x^2 + \omega_y^2}/\omega_z$ . Therefore, the reduced one dimensional GP equation can be formulated as

$$i \frac{\partial \bar{\psi}(z; \tau)}{\partial \tau} = \left[ -\frac{\partial^2}{\partial z^2} + v_{dw}(z) + 2v_n(z; \tau) + g_{1D}|\bar{\psi}(z; \tau)|^2 \right] \bar{\psi}(z; \tau), \quad (4)$$

where  $v_{dw} = z^2 + 2v_b \exp(-z^2/q_0^2)$ , with  $v_{dw} = V_{dw}/(\hbar\omega_z)$ ,  $v_n = V_n/(\hbar\omega_z)$  and  $q_0' = q_0/l_z$ . We obtain the dimensionless interaction parameter  $g_{1D} = gNm\beta/(\pi l_z \hbar^2)$  with  $N$  the total atom number.

The wave function  $\bar{\psi}(z; \tau)$  satisfies the normalization condition  $\int_{-\infty}^{\infty} dz |\bar{\psi}(z; \tau)|^2 = 1$ . The fraction of atom number in the left and right well are  $n_L(\tau) = \int_{-\infty}^0 dz |\bar{\psi}(z; \tau)|^2$  and  $n_R(\tau) = \int_0^{\infty} dz |\bar{\psi}(z; \tau)|^2$ , respectively [30].  $\theta_L(\tau) = \arctan \frac{\int_{-\infty}^0 dz \text{Im}[\bar{\psi}(z; \tau)]\rho(z; \tau)}{\int_{-\infty}^0 dz \text{Re}[\bar{\psi}(z; \tau)]\rho(z; \tau)}$  is the phase in the left well while  $\theta_R(\tau) = \arctan \frac{\int_0^{\infty} dz \text{Im}[\bar{\psi}(z; \tau)]\rho(z; \tau)}{\int_0^{\infty} dz \text{Re}[\bar{\psi}(z; \tau)]\rho(z; \tau)}$  in the right well with the density  $\rho(z; \tau) = \bar{\psi}^*(z; \tau)\bar{\psi}(z; \tau)$ .

$\phi_+(z)$  and  $\phi_-(z)$  represent the initial ground state and the first excited state wave functions for the condensates in the double-well. Their linear combinations can be defined as the left (right) mode:  $\psi_{L,R}(z) = \frac{\phi_+(z) \pm \phi_-(z)}{2}$ . They satisfy the orthogonal condition  $\int dz \psi_L(z) \psi_R(z) = 0$ . The trial wave function for obtaining the initial state can be chosen as the superposition of  $\psi_L(z)$  and  $\psi_R(z)$  as in Ref. [30]  $\bar{\psi}(z; \tau) = \psi_L(\tau)\phi_L(z) + \psi_R(\tau)\phi_R(z)$ , where  $\psi_{L(R)}(\tau) = \sqrt{n_{L(R)}(\tau)} e^{i\theta_{L(R)}(\tau)}$ . The population imbalance and relative phase at time  $\tau$  are defined as  $\Delta n = n_L - n_R$  and  $\Delta\theta = \theta_L - \theta_R$ . A given trial at  $\tau = 0$  is  $\bar{\psi}(z; 0) = e^{i\Delta\theta(0)} \sqrt{n_L(0)} \psi_L(z) + \sqrt{n_R(0)} \psi_R(z)$ .  $\Delta n(0) = n_L(0) - n_R(0)$  represents the initial population imbalance and  $\Delta\theta(0)$  the phase difference of condensates in double well. The relative phase can be measured by the interfere patterns of releasing the Bose-Einstein condensates from the double-well potential after different evolution times [13]. Moreover, a technique based on stimulated light scattering has been developed to detect  $\Delta\theta$  nondestructively [31].

The reduced time-dependent GP equation can be solved using the Split-Step Crank-Nicolson scheme, with both the space and time being discretized [32]. In our calculation, time step is  $\delta\tau = 0.001$  and space step is  $\delta z = 0.01$ . The initial ground state and the first excited state wave functions,  $\phi_+(z)$  and  $\phi_-(z)$ , can be obtained by the imaginary-time propagation. The dynamical evo-

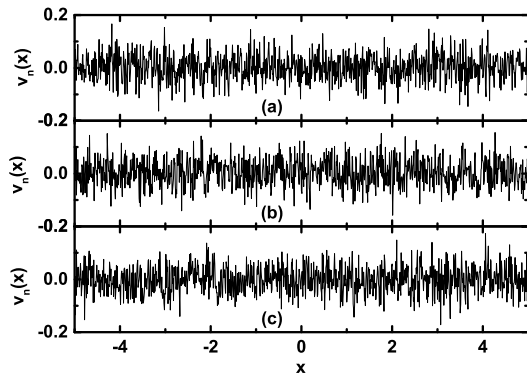


FIG. 1: Snapshots of the random Gaussian white noise potential with the noise strength  $D = 0.005$  at the time  $\tau = 1$  (a),  $\tau = 2$  (b), and  $\tau = 10$  (c), respectively.

lution are calculated by solving Eq. (4) using real-time propagation method.

The Gaussian white noise is produced numerically by the Box-Mueller algorithm [33],

$$v_n = \sqrt{-4D \ln(a)} \cos(2\pi b), \quad (5)$$

where  $a$  and  $b$  are two uniformly distributed random numbers on an unit interval, and the dimensionless parameter  $D = D_0/(\hbar\omega_z)^2$ .  $v_n$  varies occasionally from time to time. Figure 1 displays snapshots of the spatial form of the random potential at different given time. Suppose that it changes  $R$  times each unit time. Then  $R$  defines a character parameter with respect to the velocity of noise and it is called the change rate of noise hereinafter. The fastest noise is the one whose step interval  $\Delta t$  just amounts to the calculation time step  $\delta t$ . It means that the noise potential changes once per calculation step. Since the time step  $\delta\tau = 0.001$ , the change rate of the fastest noise in our study is  $R = 1000$ .

### III. RESULTS AND DISCUSSIONS

The phase-space diagram of double-well BECs without noise has been studied based on the time-dependent GP equation [20] and the obtained results are consistent quantitatively with the two-mode model results [3]. Three typical dynamical regimes can be present. (i) The Josephson oscillation regime consists of a cluster of close orbits circling the lowest energy point ( $\Delta n = 0, \Delta\theta = 0$ ) or ( $\Delta n = 0, \Delta\theta = 2k\pi$ ) where  $k$  is an integer. (ii) The  $\pi$ -mode MQST regime consists of a cluster of close orbits circling the highest energy point ( $\Delta n \neq 0, \Delta\theta = (2k \pm 1)\pi$ ). (iii) The running-phase MQST regime consists of open orbits which lie between the above two regimes.

To obtain the numerical results of dynamical double-well condensate subject to the Gaussian white noise, we set the double-well potential in Eq. (4) to be  $v_{dw}(z) =$

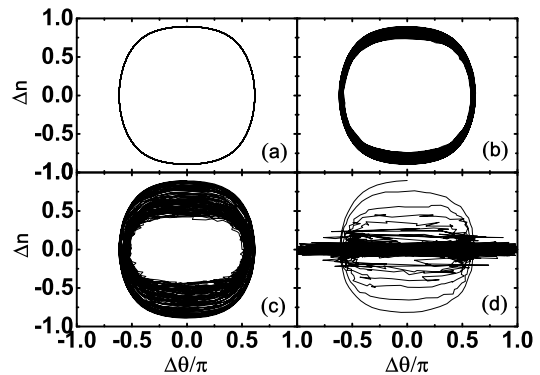


FIG. 2: Evolution trajectories in  $\Delta\theta$ - $\Delta n$  phase space of a system initially in the Josephson oscillation state ( $\Delta n = 0.9$ ,  $\Delta\theta = 0$ ) with the fixed change rate of noise,  $R = 1000$ . The evolution time is from  $\tau = 0$  to  $\tau = 5000$  and the noise strength is  $D = 0$  (a),  $D = 0.005$  (b),  $D = 0.02$  (c), and  $D = 0.5$  (d), respectively. The black horizontal band in panel d consists of many almost vertical lines and it denotes the irregularly energy-fluctuating states in the final evolution stage.

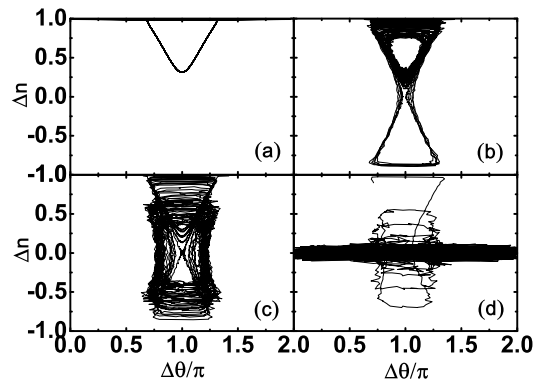


FIG. 3: Evolution trajectories in  $\Delta\theta$ - $\Delta n$  phase space of a system initially in the running phase MQST state ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.7\pi$ ) with the fixed change rate of noise,  $R = 1000$ . The evolution time is from  $\tau = 0$  to  $\tau = 5000$  and the noise strength is  $D = 0$  (a),  $D = 0.005$  (b),  $D = 0.02$  (c), and  $D = 0.5$  (d), respectively. The black horizontal band in panel d describes the same evolution stage as in Fig. 2(d).

$z^2 + 10\exp(-z^2)$  for simplicity. The interatomic interaction  $g_{1D}=0.01$ . Under above choice of relevant parameters, all three dynamical regimes appear in the phase diagram. First, we reproduce the phase diagram without noise,  $D = 0$ , and determine the location of each typical dynamical regime. Then we choose one initial state from each regime by setting the initial particle imbalance  $\Delta n$  and relative phase  $\Delta\theta$  appropriately.

We first consider the dynamical evolution subject to Gaussian white noise with high change rate of  $R = 1000$ . Fig. 2 (a) shows the orbits presented by constant energy lines for an initial state of Josephson oscillation ( $\Delta n=0.9$ ,  $\Delta\theta=0$ ) without noise ( $D = 0$ ). Both the population imbalance and the relative phase oscillate around the zero point ( $\Delta\theta = 0, \Delta n = 0$ ) during the evolution. When

Gaussian white noise with the strength  $D = 0.005$  is imposed on, as illustrated in Fig. 2(b), the trajectory gets a little “fat”, but dynamical properties are not qualitatively different from the system without noise. However, the noise effect shows up apparently when  $D = 0.02$ , as seen in Fig. 2(c). The system evolves gradually along an inward spiral path which is still smooth at earlier stage but then becomes zigzag after a long time. During this period, the system undergoes energy dissipation. Fig. 2(d) shows that the energy dissipates more quickly as the noise becomes extremely stronger ( $D = 0.5$ ). At last, the original Josephson oscillation has been completely destroyed and the system enters into a state in the black horizontal band along the  $\Delta n = 0$  line indicated in Fig. 2(d). The particle imbalance  $\Delta n$  is almost zero which means that particles tend to distribute almost equally in each well, while the relative phase varies very fast, signaling that  $\Delta\theta$  is no longer a well-defined parameter. It is also worth noting that the energy does not decrease monotonically any longer, but becomes irregularly fluctuating. Therefore, such a state is called the irregularly energy-fluctuating state in this paper. And it is also the final state of the system after a long enough evolution, no matter how weak the noise strength is.

Fig. 3 and 4 demonstrate situations of the initial state being in the running phase MQST regime ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.7\pi$ ) and  $\pi$ -mode MQST ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.9\pi$ ), respectively. In this case, each MQST regime splits into two separate parts which are symmetric with respect to the  $\Delta n = 0$  line in the phase space.

Look at the running phase MQST case. When the system is subject to a weak noise, the phase may run for a few periods and then move around a energy maximum point whose evolution trajectory looks like the  $\pi$ -mode MQST. If the noise strength turns stronger, the system starts  $\pi$ -mode-like oscillating from very beginning, as shown in Fig. 3(b). One interesting phenomenon is that the trajectory may pass across the  $\Delta n = 0$  line and then evolves around the down energy-maximum point ( $\Delta n < 0$ ,  $\Delta\theta = \pi$ ). This process will be accelerated if  $D$  is increased and the system may be in a Josephson-like oscillation around the point ( $\Delta n = 0$ ,  $\Delta\theta = \pi$ ), as shown in Fig. 3(c). The energy decreases with time in an oscillatory manner. In case of extremely strong noise, the system falls down to the irregularly energy-fluctuating state after a short period of damping.

Fig. 4 displays the cases evolving initially from the  $\pi$ -mode MQST state ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.9\pi$ ). It is obvious that the  $\pi$ -mode MQST state is more sensitive to the noise. As shown in Fig. 4(b), with the participation of noise as weak as  $D = 0.005$ , the previous closed orbit has already been becoming very fuzzy. When the noise strength is increased to  $D = 0.02$ , the system gets into running phase regime quickly. The following evolution can be regarded as a process starting from a running phase MQST state. The basic features should be similar to Fig. 3. The actual evolution trajectory depends on the concrete initial state. This is the reason why Fig. 4(c)

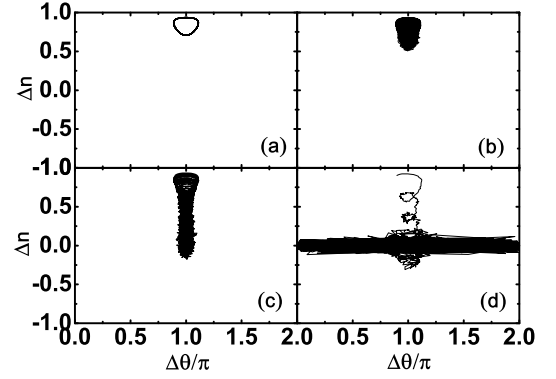


FIG. 4: Evolution trajectories in  $\Delta\theta$ - $\Delta n$  phase space of a system initially in the  $\pi$ -mode MQST state ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.9\pi$ ) with the fixed change rate of noise,  $R = 1000$ . The evolution time is from  $\tau = 0$  to  $\tau = 5000$  and the noise strength is  $D = 0$  (a),  $D = 0.005$  (b),  $D = 0.02$  (c), and  $D = 0.5$  (d), respectively. The black horizontal band in panel d describes the same evolution stage as in Fig. 2(d).

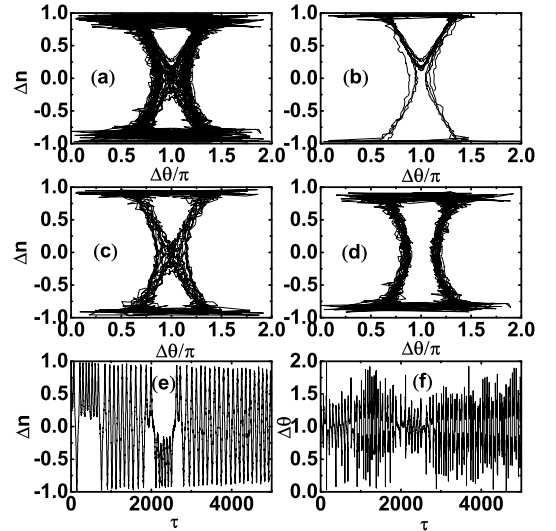


FIG. 5: Evolutions of an initially running phase MQST state ( $\Delta n = 0.9$ ,  $\Delta\theta = 0.7\pi$ ) in presence of noise with the noise strength  $D = 0.005$  and the change rate  $R = 10$ . The trajectory is plotted for the evolution time from  $\tau = 0$  to  $\tau = 5000$  (a), from  $\tau = 1000$  to  $\tau = 3000$  (b), from  $\tau = 3000$  to  $\tau = 5000$  (c), and from  $\tau = 3000$  to  $\tau = 5000$  (d), respectively. The  $\Delta n$ - $\tau$  plot (e) and  $\Delta\theta$ - $\tau$  plot (f) are shown in the third row.

and 4(d) look quite different from Fig. 3(c) and 3(d).

According to discussions above, when the noise strength is small, the Gaussian white noise may lead to an energy dissipation effect. The noise with a larger  $D$  makes the energy dissipating faster. But there are some differences in the dissipative process between the present study and the results obtained within phenomenological theories [19, 20]. The main difference lies in the evolutions from MQST states. The system damps directly towards the saddle point ( $\Delta n = 0$ ,  $\Delta\theta = \pi$ ), not the energy minimum ( $\Delta n = 0$ ,  $\Delta\theta = 0$ ). Then it steps into the irreg-

ularly energy-fluctuating state. The phenomenological theories suggest that the energy decreases all the time until the system approaches the energy minimum point. No matter how strong the dissipation is, the system will finally damp into the energy minimum state, then the evolution stops. The irregularly energy-fluctuating state does not appear.

We have calculated the dynamical behaviors of the double-well condensate subject to Gaussian white noise with different strength  $D$  and change rate  $R$ . The noise effect is mainly determined by the strength  $D$  when the change rate  $R$  is large enough. Nevertheless, the change rate  $R$  also plays an important role in understanding the dynamical evolution when  $R$  is small.

Figure 5 displays a typical case of the noise with small strength ( $D = 0.005$ ) and slow change rate ( $R = 10$ ). The evolution is from an initial state of running phase MQST ( $\Delta n=0.9$ ,  $\Delta\theta=0.7\pi$ ). Figure 5(b) looks like Fig. 3(b), but the evolution time is only within  $\tau = 1000$ , far shorter than  $\tau = 5000$  as in Fig. 3(b). Figure 5(d) shows the trajectory with the time from  $\tau = 3000$  to  $\tau = 5000$ . During this period, the system oscillates in a Josephson-like way around the saddle point ( $\Delta n=0$ ,  $\Delta\theta=\pi$ ), similar to the case shown in Fig. 3(c). It seems that decreasing the change rate gives rise to similar effect as increasing the noise strength. The  $\Delta\theta$ - $\tau$  and  $\Delta n$ - $\tau$  plots shown in Fig. 5(e) and (f) provide detailed information of the evolution process. Atoms can be trapped almost in one well for a while and then oscillate between the two wells. The variation range of  $\Delta\theta$  is significantly enlarged and many extremely sharp peaks appear in the  $\Delta\theta$ - $\tau$  line. These peaks indicate that the relative phase can change extremely fast, which may signal that the relative phase is no longer well-defined, as discussed above. Such situation occurs preferentially in the large  $\Delta n$  region, as indicated by the almost vertical lines in Fig. 5(a). This is different from the large  $D$  case, in which the almost vertical lines appear in the  $\Delta n = 0$  region first. If the system evolves further, more almost vertical lines appear and they tend to cover the whole phase space in an irregular manner. As a result, it is almost impossible to expect in which state the system is at a given time. Meantime, the energy of the system does not simply tend to damping, but may go up and down occasionally. So the energy dissipation effect seems smeared out during this stage.

#### IV. CONCLUSION

In conclusion, we have investigated how an external Gaussian white noise affects the dynamics of condensates

in double-well. Dynamical evolutions from three typical dynamical regimes of Josephson oscillation,  $\pi$ -mode MQST and running phase MQST are discussed. It is shown that the system keeps its original dynamical feature for a long time when it is subject to a noise with weak strength and rapid change rate which we called the weak noise. In this case, the noise induced energy dissipation effect is observed. Energy of the system decreases with evolution monotonously in the Josephson oscillation regimes, and oscillatorily in the MQST regimes. The difference between results of the present study and the phenomenological theory is discussed.

Either increasing the noise strength  $D$ , or decreasing the noise change rate  $R$  may give rise to significant influence on dynamics of the system. If  $R$  is large enough, the influence of  $D$  is quite clear. The larger  $D$  is, the faster the state evolves and damps. Moreover, we also figure out that the noise with large strength drives the system finally into the irregularly energy-fluctuating state, in which the population imbalance tends to being very small and the relative phase is no longer well-defined. This phenomenon is different from the dissipation effect which makes the condensates evolve from the high energy state to the lowest energy state where the particle imbalance is zero and the relative phase is  $2k\pi$  ( $k$  is an integer). Decreasing  $R$  brings about similar effect as increasing  $D$  in the early stage of evolution. After a period of time, the noise with slow change rate compels system to change irregularly, with the relative phase being destroyed. But this phenomenon happens preferentially in the large  $\Delta n$  region in presence of the noise with slow change rate, which is different from the effect caused by the noise with large noise strength but fast change rate.

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